

## Tetrahedral Lattice Theory of Quantized Space-time and the Experimental Determination of Its Fundamental Parameters

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### Abstract

A Tetrahedral Lattice theory of Quantized Space-Time (TLS theory) is presented. The logical and observational justification for its need is presented. The theory determines the fundamental quantities of quantum of distance ( $\delta$ ), area ( $\alpha = (3/8)3^{1/2}\delta^2$ ), volume ( $\varphi = (1/8)3^{1/2}\delta^3$ ), and time ( $\tau$ ). An experimental procedure for the determination of these fundamental quantities is presented. Using the known, experimentally obtained list of the wavelengths of emission and absorption spectra of the elements, applying the before mentioned procedure, the fundamental distance is calculated. From this value, the other fundamental quantities are calculated. The values obtained are:  $\delta = 5.0 \times 10^{-33}$  cm,  $\alpha = 1.6238 \times 10^{-65}$  cm<sup>2</sup>,  $\varphi = 2.7063 \times 10^{-98}$  cm<sup>3</sup>, and  $\tau = 1.6667 \times 10^{-43}$  sec.

**Keywords:** Discrete Space, Plank distance, Space-time topology

### Introduction

It was in 1973, when attending to the first semester of Calculus at the university, that I had my first encounter with the problem of continuity and Zeno's paradox. The professor struggled with some examples to make us students understand something that not even he himself could understand fully; I entered into a hot argument with him, finally, to convince him of my point I told him that if he were standing in front of a wall against a shooting squad he would know that, once the soldiers shoot, the bullets would reach and pass him. From that moment on two things happened to me: I was in the professor's sight for shooting, and never again would I leave this problem alone until I resolved it. It was in the summer of that same year when I understood the problem, and from that understanding came the solution.

#### Zeno and the Paradox of Motion<sup>1</sup>

Zeno of Elea was an ancient Greek (born around 490 B.C.) who lived in what is now southern Italy. He was a disciple of Parmenides, a philosopher that maintained that reality is singular and unchangeable, and that the perceptions of plurality and motion are illusions. Zeno is best remembered for a series of arguments called Zeno's paradoxes.

Scholars are not sure about Zeno's intentions with his paradoxes; some think that he wanted to demonstrate that motion is impossible, but we may surely conclude from his

paradoxes that Zeno purported to prove, by logical means, that any explanation of plurality and motion, essential features of the perceptual universe, results in a contradiction. This contradiction for him would demonstrate that the perception of things is an illusion, and that there is a singular and unchanging reality behind the appearance of the universe.

Although it cannot be known for sure how Zeno himself viewed his "paradoxes", we can nevertheless examine the arguments themselves, as they have come down to us. Of his paradoxes, the one directly related to the purpose of this paper is the first paradox of motion: the dichotomy.

*The dichotomy* asserts the non-existence of motion because what moves must pass through an intermediate position between the starting point and the final destination before it arrives at the end, and so on ad infinitum for each step it takes. This paradox is known as the 'dichotomy' because it involves repeated division (partitioning) into two.

Some scholars try to "solve" this paradox by pointing out that an infinite series can have a finite sum. But this solution is fallacious, because to say that an infinite series can have a finite sum is not the same thing as traversing a path that have an infinite amount of steps. If you have to go through all the points of the series, you will never reach the end, even if you have a series that converges to a limit, and the distance is finite. The question to answer here is: If you have to pass through every position in the path between any two positions before arriving at the next one, when do you take the first step? The logical answer is never, because there will always be an infinite number of positions between the actual position and the next one.

In order to understand the problem, we have to unveil the hidden assumptions behind the paradox. The basic assumptions made in the Dichotomy are:

1. space is continuous, that is, that any spatial interval is composed of an infinite number of dimensionless points; and
2. motion is continuous, that is, any change of position takes place only passing through all the positions between the starting and the end points. Having cleared

this, we may analyze the paradox. To do this, let us arrange the dichotomy in the form of a logical argument.

- a) The axiom is: Any distance is a spatial interval that is composed of an infinite number of points (Space is continuous).
- b) In order to move from the starting position to the final one of any spatial interval you have to go through all intermediate points (motion is continuous). Since you could have discrete motion in a continuous space, this is not necessarily a forcibly concomitant condition.
- c) Since the number of intermediate points between any two points in the interval is infinite, you will never reach the end, because you will never be able to take the first step and reach the next point in the series.
- d) Therefore, such a movement is impossible.

This reconstruction of the argument is valid. If space and time are continuous, then the inevitable conclusion is that motion is impossible. Nevertheless, since we accept motion as a given, we must question the premises. The only way out of the paradox is to conclude that the premises presented above are, either both false, or space is not continuous. But, what happens if we assume that space is discrete? Let us rearrange the syllogism, taking into account this new axiom, to see the results.

- a) Any distance is a spatial interval that is composed of a finite number of indivisible units, being each unit, a position in space.
- b) In order to move from the initial position to the final one of the spatial interval you have to go through all intermediate spatial positions (in this case motion cannot be continuous because it occurs in a discrete medium).
- c) Since the number of intermediate intervals is finite, you will reach the end.
- d) Therefore, you can move.

Seeing Zeno's argument on motion with this formal analysis, we arrive at the conclusion that either space is discrete or motion is impossible. Even an illusion requires an illusionary space to take place in, thus even if motion is an illusion, it requires space to be discrete in order to take place. The only inevitable conclusion is: Reality and for that matter space are discrete.

You may argue that you could have discrete motion in a continuous space, thus avoiding the necessity of passing through all intermediate positions, but if we take into account the implicit definition of distance (Any distance in a continuous space is a spatial interval that is composed of an infinite number of points); then in a continuous space there are only two possible distances: zero, when the starting and final points coincide; and infinite, when the starting and the final point are different; which is the same as saying that continuous space is a singularity where distances and positions cannot be distinguished, making any change of position (motion) impossible. Thus the final

conclusion is that space must be discrete, and by the same token so is motion.

## Quantized Space-Time Theory

### Previews encounters with the problem

The concept of discrete space and time (or chronon) has a long history (Snyder<sup>2</sup>, Caldirola<sup>3</sup>, Wolf<sup>4</sup>, and Wolfram<sup>5</sup>). The idea that matter might be made up of discrete particles existed in antiquity; already around 450 BC it was discussed by Leucippus and Democritus<sup>6</sup>, and occasionally the notion was discussed that space might also be discrete - and that this might for example be a way of avoiding issues like Zeno's paradox.

In 1644 René Descartes<sup>7</sup> proposed that space might initially consist of an array of identical tiny discrete spheres, with motion then occurring through chains of these spheres going around in vortices - albeit with pieces being abraded off. But with the rise of calculus in the 1700s all serious fundamental models in physics began to assume continuous space. In discussing the notion of curved space, Bernhard Riemann<sup>8</sup> remarked in 1854 that it would be easier to give a general mathematical definition of distance if space were discrete.

But at that time physical theories required continuous space because space was assumed axiomatically continuous, thus the necessary new mathematics was developed and almost universally used. But still by 1887 William Thomson<sup>9</sup> (Lord Kelvin) did consider a discrete foam-like model for the ether.

Starting in 1930, difficulties with infinities in quantum field theory again led to a series of proposals that space-time might be discrete<sup>10</sup>. And indeed by the late 1930s this notion was fairly widely discussed as a possible inevitable feature of quantum mechanics. But there were problems with relativistic invariance.

### The Heisenberg Case

As of 1929, in an attempt to solve the problems of the infinities that arise with the self energy of charged particles, Heisenberg presented the idea of turning the world into a big lattice; that space itself might be construed as a honeycomb of minute cubic cells of the size of an elementary particle. If such cells existed, they would constitute an absolutely minimum distance, below which the size of any elementary particle, or any size whatsoever, could not shrink. He argued that such minimum distance would make infinitesimal particles impossible, thus rendering self-energy finite<sup>11</sup>. Although the argument for this idea is correct, the idea itself is founded, as we demonstrate below, in wrong mechanical and Euclidean concepts of space; rendering the idea as naïve.

### Renormalization

After ideas of renormalization developed in the 1940s, discrete space seemed unnecessary, and has been out of favor ever since. Some non-standard versions of quantum field theory involving discrete space did however continue to be investigated into the 1960s, and by then a few isolated other initiatives had arisen that involved discrete space.

## Revival of the Concept of Discrete Space-Time

The idea that space might be defined by some sort of causal network of discrete elementary quantum events arose in various forms in work by Carl von Weizsäcker<sup>12</sup> (ur-theory), John Wheeler<sup>13</sup> (pregeometry), David Finkelstein<sup>14</sup> (space-time code), David Bohm<sup>15</sup> (topochronology) and Roger Penrose<sup>16</sup> (spin networks). General arguments for discrete space were also sometimes made - notably by Edward Fredkin<sup>17</sup>, Marvin Minsky<sup>18</sup> and to some extent Richard Feynman<sup>19</sup>, on the basis of analogies to computers and in particular the idea that a given region of space should contain only a finite amount of information.

In the 1980's approximation schemes such as lattice gauge theory and later Regge calculus<sup>20</sup> that take space to be discrete became popular, and it was occasionally suggested that versions of these could be exact models. There have been a variety of continuing initiatives that involve discrete space, with names like combinatorial physics - but most have used essentially mechanistic models<sup>21</sup>, and none have achieved significant mainstream acceptance. Work on quantum gravity in the late 1980's and 1990's led to renewed interest in the microscopic features of space-time<sup>22</sup>. Models that involve discreteness have been proposed (most often based on spin networks) but there is usually still some form of continuous averaging present<sup>23</sup>, leading for example to suggestions that perhaps this could lead to the traditional continuum description through some analog of the wave-particle duality of elementary quantum mechanics.

In "Three Roads to Quantum Gravity" Smolin<sup>24</sup> summarizes that black hole thermodynamics, loop quantum gravity and string theory each takes a different starting point, but they agree in that when viewed on the Planck scale ( $10^{-33}$  cm), space and time cannot be continuous,

Loop quantum gravity gives us a detailed picture of the units of space, in terms of spin networks. It tells us that volumes and areas are quantized and come only in discrete units. String theory at first appears to describe a continuous string in a continuous space. But a closer look reveals that a string is actually made of discrete pieces called string bits, each of which carries a discrete amount of momentum and energy. Black hole thermodynamics leads to an even more extreme conclusion, the Bekenstein bound. According to this principle the amount of information that can be contained in any region is not only finite, it is proportional to the area of the boundary of the region, measured in Planck units. This implies that the world must be discrete on the Planck scale, for were it continuous any region could contain an infinite amount of information.

For seemingly different reasons, at the end of each of these roads one reaches the conclusion that the old picture according to which space and time are continuous must be abandoned. On the Planck scale, space appears to be composed of fundamental discrete units. It is the basic conception of space and time which has to be changed, and then it is from this new picture of these fundamental aspects of existence that the fundamental concepts on which physics is based should be reviewed and reconstructed. The same reason that the equations of electric engineering do not apply in the atomic realm because they assume a continuous

distribution of charge (which in reality is discrete by  $1.620 \times 10^{-19}$  coulombs), the theories of physics applied to macro-phenomena do not apply when continuity of space is no longer apparent because the quantities been measured are too close in order of magnitude to the realm where the inner structure of space is revealed.

## Quantized Space Topology

As we have seen, present theories, including quantum mechanics and general relativity, are based on the assumption of continuous space-time. In these theories, it is implicitly assumed that space-time is continuous and infinitely divisible. But whether space-time is continuous or not, is up until now an unsolved problem. In fact, the proper combination of quantum mechanics and general relativity has implied the existence of discrete space-time, in which the minimum space and time unit is called respectively Planck length and Planck time<sup>25</sup>. Recently, motivated by the latest direct detection of time asymmetry in Kaon decay at CERN and Fermilab, it has been suggested a theoretical rationale for this puzzle, in terms of quantized time<sup>26</sup>

The problem of the implied necessity of the discreteness of space-time at Planck's scale is not solved by stating that space and time are discrete, we must construct a physical and topological theory of discrete space and time that would not contradict observation and would be logically and conceptually consistent with the whole body of theoretical and experimental physics. For such a theory two things are of the utmost importance:

1. a topology of physical space that fulfils all requirements as not to contradict physics theory and observation, and
2. to find an empirical (experimental) reliable method to measure the fundamental quantities implied at the basis of the theory.

For such measurements approximations will not suffice, because the theory should point to undisputable experimental procedures to measure the fundamental quantities it postulates, the only errors admitted are experimental errors.

We do not postulate physical space here as the container of the stuff called matter, but the result of the fact that the basic units of that stuff exist as an ordered set, and as such relate to each other generating a configurational structure or relational space. It is simple to realize that if there were only one single basic stuff the relational space would be void and nothing of what we recognize (no matter whether it is real or imagined) would have any existence. So we can certainly conclude that the basis of matter is manifold and because of that space exists.

If space is but the configurational structure of the set of "real things"; two aspects must be considered in order to propose a theory of the basic structure of space:

1. What is the inner topology of those basic "real things" that would allow to determine the manner in which they would interact (or relate to each other) to

develop predetermined configuration patterns? (Only predetermined sets of configuration patterns would be allowed).

2. What would be the basic topological structure of the allowed configurational structures that emerge from the “interaction” of the basic “real things”?

These two conditions must be sufficient to explain the emergence of “compound” entities and their interaction rules (a compound entity being a specific configurational structure); as well as the observed macroscopic properties of space (topology, non-locality, quantification: distance, area, volume; and other unforeseen properties) and finally the emergence of time.

Here we see another reasoning pointing at the necessity to conceive space as discrete. If physical space is the relational structure of the basic elementary stuff of which matter is made, and if those basic elementary things are finite in terms of the number of types and their total number in the universe (which implies that their set is discrete), then the relational (or configurational) space they generate is also finite and discrete; so physical space must be discrete and finite.

A very interesting feature that this idea of space generates is that, since there are different types of relations between the entities (including the type of relations called interactions), the interaction of entities generates new entities, which in turn generate new types of relations (including interactions) generating a propagating growth of the number (in kind and total amount) of relations (configurations); this implies that physical space must be continually exponentially expanding. The closest concept to this idea of space we found in the literature is Barbour’s Configurational Space<sup>27</sup>.

The discussion on the foundations of a theory of physical space and time is beyond the scope of this paper and will be treated in another publication. But some important conclusions must be put forward.

The so called permitted configurations will entail a network of nodes (nodes are just positions in the network) which in the end would be the Space Lattice.

Assuming that there is a minimum number of relations (or interactions) that occur simultaneously between fundamental entities, then every node in the net will have at least this number of connections with neighboring nodes, this would make every node to be a template (as defined by Wolfram) with a rule of connectivity that would imply this minimum number. This kind of templates cannot be reduced to a node with three connections, as in the case of Wolfram’s templates, because of the restrictions imposed by the rules.

This is equivalent to saying that the nodes, instead of being dots with simple line connections between them, are geometrical solids in which the connectivity between them is made through their “faces” instead of lines. The number of sides of the faces would be equal to the number of concerted interactions between the entities.

If we form a Lattice Space with the network of the basic minimum interactions between the fundamental entities,

this would be the background behind which all observed phenomena (complex emerging and basic interactions) will occur. Taking as constraints the macroscopic properties of physical space it is possible to deduce the required topological restrictions on the geometry of the templates (solid geometric bodies) and their face to face network to form the lattice; that is, the minimum number of nodes the template would have (minimum number fundamental interactions), and the minimum number of concerted fundamental interactions that occur between fundamental entities (the geometry of the faces, how many sides would have the faces; because of the observed isotropic property of physical space only regular geometric bodies are allowed).

The fact that there is a minimum number of kinds of relations by which fundamental entities can interact, implies that there cannot be purely electrical nor magnetic interactions, nor purely gravitational, strong, or weak interactions; it means that every time fundamental entities interact a minimum number of fundamental interactions must take place simultaneously in a concerted way; in the Interaction Space Lattice they will be orthogonal.

If all we can observe is the fruit of interactions, then the concept of Empty Space is nonsense; vacuum simply means the absence of massive particulate bodies (which are the statistical result of certain interaction configurations).

### Discrete Physical Space Topology

For the reasons already explained, the topology for the discrete physical space will be assumed to be a lattice topology of solid basic unit cells. For a topology of physical space to fulfill all the above mentioned requirements it has to contemplate the following characteristics:

1. **The topology must be “All-filling”.** That is, the unit cell must be of such geometric characteristics that the packing to form the lattice would not form interstices; otherwise that space would have two or more domains, the one formed by the unit cells of the lattice, and the one formed by the interstices. One example of the kind of topology that is not All-filling is the one formed by the packing of spheres; here you have two separate domains: 1) the one formed by the spheres, and 2) the one formed by the spaces surrounded by four spheres. An object could be said to exist in the domain of the interstices (for example a liquid filling the spaces between solid balls), or in the domains of the spheres (for example mechanical vibrations being transmitted across the lattice formed by the solid balls). We could say that a physical space with such a topology has more than one existential dimension; the type of objects that exist in one of them not necessarily could exist in the other; because the topology of each domain, being different, determines the type of objects that could exist in it and the type of “behavior” of the objects proper to it. In other words, the two or more domains of the topology of such a space would be like parallel universes that cannot interact, and the one would not know of the other. We conclude that not all geometrical shapes would suit the necessary requirements to be the unit cell of a quantized physical space topology.

- 2. The unit cell must be final.** That is, the geometry of the unit cell must be such that you cannot fill a unit cell with smaller unit sub-cells of the same type but proportionally smaller. This requirement is necessary because otherwise you will fall into the predicament of continuity, because there is no justification to say where we stop in finding the finer grain of space, there will be always a smaller grain that still will produce an All-filling topology if the geometry of the unit cells does not have this restriction. There must be a topological reason for a unit cell of a lattice to be the final and only unit cell, the most obvious reason is that you cannot fill the unit cells with “smaller” but equal in shape sub-cells and still produce an All-filling topology inside the cell. The cube is a good example of what geometry is not proper to be the unit cell of a discrete physical space topology, because, as can be easily seen, it does not fulfill the requirement of being final.
- 3. The lattice must be isotropic.** The lattice must be such that no region of it could be distinguished in any way, other than relative position in the same lattice, from any other region in the lattice. Otherwise, whatever element contained in the lattice will have different properties and “behavior” in those differing regions. Another important aspect of this requirement is that in terms of directionality the lattice must also be isotropic, that is, there must not be preferential directions for a step-wise change of position in the lattice. Isotropy is an observed property of physical space, so this requirement is necessary in order not to contradict the empirical observation. What this requirement indicates is that 1) the unit cells of the lattices must be of one kind and size only (one geometry and one dimension), and 2) the packing must be so that there is no preferential direction to form the lattice. From this we conclude that no parallelogram could be the geometry of the unit cell of the quantized physical space.

The problem of packing of solid geometrical bodies has been studied throughout the centuries<sup>29, 30</sup>. After struggling with misconceptions and cunning approaches, a thorough understanding of the problem has already emerged. But, the necessary restrictions imposed on the topology of the physical space matrix for it to be consistent both, with the basis of the relational structure that underlies it, and with the observed macroscopic characteristics of this space, are all of topological, not of geometrical nature (see above). The problem of packing of solids is restricted to the geometry of packing of geometrical solids defined in a Euclidian space; the arrangements of these solids as to not to leave empty regions of space (Euclidean) is what has been explored, determining, for each arrangement, only two parameters: 1) the percentage of space filled, and 2) the angular “error” implied when the “all filling” condition is not attained

So, when we are talking about the need for an “all filling” topology, we are not referring to a geometrical requirement, nor imposing any geometry to the physical space matrix and its “unit cells”. What is meant is that the topological connectedness between the unit cells to form the matrix must be of such nature as to prohibit the possibility for the existence of a “not connected” region of the matrix.

Thus, the tetrahedron is the only regular geometric solid that fulfills all the topological requirements mentioned above. Historically the tetrahedron confused mathematicians as to fulfilling also the requirements for Euclidean space, but the issue has been already clarified<sup>29</sup>. This confusion lasted centuries because the logical-mathematical tools necessary to clarify the matters were not developed up until recently<sup>29, 30</sup>, and because the angular deviation is so minute that it misguided intuition<sup>31</sup>. It is good to keep in mind that modern physics has shown physical space to deviate from Euclidean space both, at Planck and cosmological scales<sup>32</sup>; and that diamond is a tetrahedral crystalline lattice without a crack (i.e. fulfilling the “all filling” requirements) imbedded in physical space; knowing that Euclidean space does not allow for that situation is another proof that physical space is not Euclidean.

This means that physical space must have properties that agree with the topology of a tetrahedral lattice. In the same way the properties of elementary and complex objects that exist in physical space must in the final analysis be explained in terms of this topology.

If the topology of the physical space is that of a Tetrahedral Lattice, in virtue of the theory of space here proposed, we may conclude that 1) there are four fundamental interactions (the vertices of the tetrahedron), and 2) that at the fundamental level, interactions occur in groups of three (the faces connecting the tetrahedrons in the lattice). In the following paragraphs we analyze the topology of a Tetrahedral Lattice Space (TLS), in which the unit cells are solid regular tetrahedrons.

The Basic units of physical space are not simple dots without dimension, but unit volumes. As we have seen, these unit volumes have a tetrahedral topology. The basic geometry of the regular tetrahedron is determined by its edges. The geometry of the tetrahedron is better suited for our purposes when we study it defining another geometrical parameter, i.e. the radius ( $\rho$ ); the radius is defined as the Euclidian distance from the geometrical center of the solid tetrahedron to the geometrical center of any of its faces. This distance is normal to each face of the tetrahedron. It can be shown that the radius is related to the edge by the simple relationship  $\rho = 1/2 (2/3)^{1/2} a$ , where  $a$  is the length of the edge. Thus, the fundamental unit volume ( $\varphi$ ) of space is the volume of a tetrahedron which radius is a fundamental parameter to be determined. The unit area ( $\alpha$ ) would be then the area of one of the faces of the fundamental tetrahedron. These parameters, expressed in terms of the radius, are  $\varphi = (3)^{1/2} \rho^3$  and  $\alpha = (3/2)(3)^{1/2} \rho^2$ .

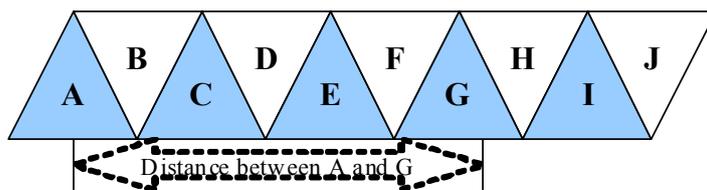
We are very much used to measure distances when what we are really measuring is volumes; no matter what sort of instrument we use, when we are speaking of the measurement of something existing out there and not a parameter that suits as an indirect measurement of the real thing, we cannot be measuring distances (a supposed string of continuous non-dimensional points that produces a one-dimensional stuff called a line) as defined in euclidian form, or as the algebraic squared root of the difference between the squared values of the two “locations” in a  $\mathbf{R}^3$  space. That type of topology does not apply here. In this context, distance should be redefined.

Since physical space is a lattice, as we have seen, every cell of the lattice (a tetrahedron) has a very well defined position with respect to all the other cells in the lattice (the topology of solid unit cells is much simpler than the topology of the void unit cells because the number of elements to relate in the topology is lower<sup>33, 34, 35, 36, 37</sup>). So we are dealing here with a well ordered set of things (the tetrahedrons), each of those elements has a well defined inner topology (i.e. each of its parts has a specific orientation to each of the other parts, and a defined metric relationship); and at the same time each element of the lattice (a tetrahedral cell) has a well defined orientation with respect to the other elements of the lattice, i.e. each tetrahedron is connected to four other tetrahedrons through each of its four faces, in such a way that the two adjacent radii of two adjacent tetrahedrons form a “diameter” equal in size to double the radius, while the “diameter” formed between the central tetrahedron and any of the adjacent tetrahedrons forms an angle of around  $109^\circ$  with the other “diameters” stemming from the central tetrahedron (See **Figure 3**); thus forming a perfect and simple packing topology, that perfectly relates the topology of the lattice to the topology of the unit cells of the lattice.

Using a simple definition of distance, for example, the one that relates the path of the minimum number of unit cells between two unit cells of the lattice, we could establish the basis for the topology that would relate the unit volume to

a distance parameter that would allow the calculation of volumes and areas in terms of this parameter. The simplest definition would be: “Distance is the cardinality of the subset defined by the path of the minimum number of elements between two specific elements of an ordered set”. In our case we have an ordered set (the lattice) and its elements are the unit tetrahedral cells. The cardinality is a plain integer with no unit to distinguish what is being counted by it; but if we add a predicate to the number that indicates what is the thing counted, then it would indicate how many of those things we are referring to instead of a simple cold number as the purely mathematical cardinality does. Thus the distance would be  $\mathbf{d} = \mathbf{m}\delta$ , where  $\mathbf{m}$  is the cardinality of the sub-set comprehended between two  $\delta$  elements, and  $\delta$  is the unit parameter of the element. This definition states how many ( $\mathbf{m}$ )  $\delta$  things are there between two other  $\delta$  things. All these roundabouts are necessary in order to avoid confusion.

This unit distances add in an arithmetical way. Even though their apparent geometry indicates that each unit distance along the path is at  $\sim 109^\circ$  out of line to each other, the distance that we calculate is not a euclidian geometrical distance. If we know that distance, we know how many unit volumes we have to travel in order to get from point **A** to point **G** (see **Figure 1**) and this is what matters in physics.



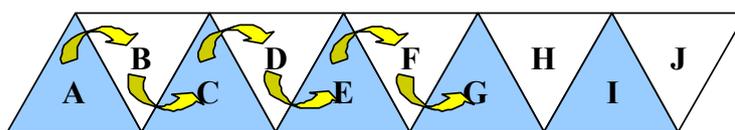
**Figure 1.** The physical distance ( $\mathbf{d}_p$ ) between **A** and **G** is the cardinality of the subset  $\{\mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}, \mathbf{F}\}$ , i.e. 5; and between **A** and **B** is 0.

In a lattice space with unit volumes two types of distances have to be defined:

**1. The Physical distance ( $\mathbf{d}_p$ ).** Physical distance is the number of positions that ought to be passed in order to arrive at **G** from **A** (see **Figure 1**). This definition clearly shows that the cardinality of the subset that lies between **A** and **G** is the proper answer. According to this definition the physical distance ( $\mathbf{d}_p$ ) between two adjacent positions (**A** and **B**) is zero, because there is no element between **A** and **B**.

**2. The Action distance ( $\mathbf{d}_a$ ).** Since there is a positional difference between two adjacent elements, there must be a distance between them, because there has to be a change of position to go from **A** to **B**.

The definition that suits this situation is “distance is the number of unitary steps necessary to go from **A** to **G** (a unitary step is equal to the change of position between two adjacent positions)”; we will call this the “action distance” ( $\mathbf{d}_a$ ), see **Figure 2**.



**Figure 2.** The action distance ( $\mathbf{d}_a$ ) between **A** and **B** is equal to the number of unitary steps necessary to change from the position at **A** to the position at **B**, i.e 1; between **A** and **G** is 6.

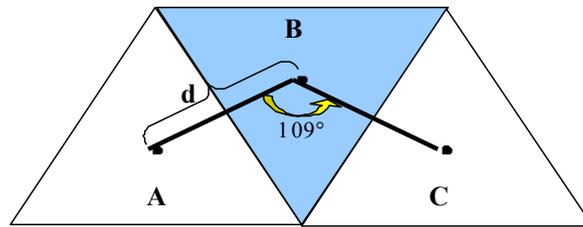
It is clear that,

$$\mathbf{d}_a = \mathbf{d}_p + 1$$

In physics all our measurements are dynamic; this is why in physics we always measure  $\mathbf{d}_a$ . As can be seen in **Figures 1**

and **2**, the proper predicate for  $\mathbf{d}_a$  is not the unit volume, because there is no volume between two adjacent positions, which would make  $\mathbf{d}_{AB} = 0$ . From this point on we will have to use the topology of the unit tetrahedral cells to determine the distance between two adjacent positions. Two adjacent positions differ 1) in orientation relative to each other, and

2) where they lie with respect to each other across the diameter that connects their two centers (See **Figure 3**). So on moving from **A** to **B** two things change: 1) the orientation of the unit cells in the lattice (The tetrahedron has to rotate in order to occupy another position; each rotation is of a magnitude of  $\sim 109^\circ$  between adjacent positions. We will call this rotation “**spin**”), and 2) the relative position along the diameter that connects them (See **Figure 3**). While the **spin** is an important feature of the change of position (motion), here we will focus on the change of relative position. This



**Figure 3.** On moving from **A** to **B** two things change: 1) the orientation of the unit cells in the lattice (The tetrahedron has to rotate in order to occupy another position each rotation is of a magnitude of  $\sim 109^\circ$  between adjacent positions. We call this rotation “**spin**”), and 2) the relative position along the diameter (**d**) that connects them (white moves to blue).

From here we can calculate the fundamental quantum of area ( $\alpha$ ) and the fundamental quantum of volume ( $\varphi$ ) in terms of the fundamental quantum distance ( $\delta$ ):

$$\alpha = \left(\frac{3}{8}\right)3^{1/2}\delta^2$$

$$\varphi = \left(\frac{1}{8}\right)3^{1/2}\delta^3$$

Defining distance correctly in a Tetrahedral Lattice Space in such a way that it allows us to know the fundamental quantum of distance and to calculate the fundamental unit of volume and area, paves the way to allow us to understand the macroscopic properties of space from its fundamental structure; but, as we have seen, in this theory distance is directly related to the capacity of changing position in the lattice (motion), i.e. the Action distance; for this reason some other aspects of space and motion are also revealed. This will require redefining other fundamental parameters that have been used or defined according to basic conceptions biased by the perceptual illusion of continuous physical space.

If we define “degree of freedom of motion” as the “total number of possible directions in which an object, positioned at certain point in space, can move”; then the degree of freedom of a free particle in a continuous physical space is infinite. Whereas in a Lattice Space, it will depend on how many adjacent positions surround the position the body is at; in a Tetrahedral Lattice Space there are only four positions. This means that at the fundamental level, that is, at the level of fundamental interaction between fundamental entities, the degree of freedom of motion of free fundamental particles is restricted to four directions; in each case the particle will have to rotate relative to the previous position in a different manner. We may define “dimension of a space” as “the number of degrees of freedom of a free object in space that cannot be expressed as a linear combination of a lesser number of degrees of freedom in the same space”. In

change of position is directly related to the diameter, a parameter of the inner topology of the unit tetrahedral cell that contains in it the unit volume of the cell. From **Figure 3** we can see that the proper predicate for  $d_a$  is the diameter (twice the radius) between two adjacent tetrahedrons ( $\delta = 2\rho$ ). Thus the distance that we measure in any experimental situation is  $d_a = m\delta$ , where  $m$  is any integer,  $\delta$  the diameter across two adjacent quanta of space is the fundamental quantum distance.

other terms, the dimensionality of a space is the base of that space. It is clearly seen that Euclidian space has three dimensions; while the Tetrahedral Lattice Space (TLS) has four dimensions, because each direction at every position is independent of the other and cannot be expressed as a linear combination of any set composed of the others.

It is apparently shocking to say that TLS has four dimensions, because it apparently contradicts the observed qualities of physical space at macroscopic level. But it is easily shown that this is not the case. Let us take for example the structure of diamond; at the atomic level diamond is formed by a tetrahedral lattice with a carbon atom at the center of each tetrahedron, each unit cell of this lattice has four faces, so to go from one carbon atom to the neighboring one there are only four possible directions; but as we increase the number of cells in a sample volume, so increases the number of faces (directions) in the surface of the sample volume. When we hold in our hands a one-gram diamond, the sample volume we are dealing with contains a little less than  $10^{23}$  atoms, each one the center of a tetrahedron. Imagine, if one tetrahedron has four faces (directions), two tetrahedrons have six (each tetrahedron you add eliminates the faces that face each other, but adds the remaining three of the added tetrahedron), and so on increasing until you add together  $10^{23}$  tetrahedrons, the number of directions (degree of freedom) is staggering, so it will seem to us as an infinite degree of freedom (just like euclidian space), so our intuition will tell us that the basis of the diamond must be three dimensional; when in reality it is not. So a tetrahedral lattice at the macroscopic level will look three dimensional and continuous (and it will work perfectly for macroscopic phenomena), but when we want to study fundamental phenomena and entities, TLS topology would be the correct tool.

### Measurement of $\delta$ and $\tau$

We have stated above that, one of the most important requirements for a theory of discrete physical space is that it should directly point at an empirical (experimental)

reliable method to measure the fundamental quantities implied at the basis of the theory. We show bellow one reliable and reproducible experimental procedure. It is easily reproducible because the data necessary for the final calculations to be performed already exists, and is published in many different scientific manuals and catalogs.

### Light propagation and Quantized Space-time ( $\delta$ and $\tau$ ).

According to Aybar's EMG Theory of the Photon, a photon has four distinct configurations which run in a series ( $\Phi_n$ , where  $n= 1,2,3, \dots, \Phi_n = \Phi_{n+4}$ ), when a photon moves from one position to the next, it changes from one configuration to the next, only quantized displacements (instantaneous quantum changes of position) are possible. According to the theory, the distance leaped by the photon is given by  $\Delta \mathbf{x} = \lambda/8$ , that is, every time the photon changes position it jumps a distance equal to 1/8 its wavelength; all the positions of the photon are determined by  $\mathbf{x} = \mathbf{x}_0 + \mathbf{n}\lambda/8$  ( $\mathbf{x}_0$  is any starting position,  $n = 1,2,3,\dots$ ). In the same way it is shown that for the photon time changes in quantized amounts ( $\Delta t = 1/8T$ ) equal to 1/8 of the photons period, so that the series of sequential time intervals for the photon's motion is given by  $\mathbf{t} = \mathbf{t}_0 + \mathbf{n}T/8$ . Since  $|\Delta \mathbf{x}/\Delta \mathbf{t}|_{\Delta n=1} = \lambda/T = \mathbf{C}$ , then  $\Delta \mathbf{x} = \mathbf{C}\Delta \mathbf{t}$ .

### Light wavelength and $\delta$ .

According to this theory of discrete space-time, any distance can be expressed as an integer multiple of the fundamental minimum distance ( $\delta$ ), that is  $\mathbf{d} = \mathbf{m}\delta$  (where  $\mathbf{m}$  is any integer). Since the photon's wavelength is a direct measurement of the distance the photon jumps during its displacement, knowing the wavelength of a photon leads to knowing a naturally occurring distance, so that if we know a series of those distances we could use it to determine the fundamental minimum distance. Thus, for every photon  $P_i$ , it holds that  $\Delta \mathbf{x}_i = \lambda_i/8 = \mathbf{m}_i\delta$ . It so happens that for a very long time such a series of well known naturally occurring wavelengths exists, and is available to everyone. This series is the list of wavelength values of the atomic emission and absorption lines of the elements. Bellow we describe the way we used that information to calculate the fundamental minimum distance and time ( $\delta$  and  $\tau$ ).

### An experimental Method for determining $\delta$ and $\tau$

In order to perform the calculation of  $\delta$  all the tables listing the known emission and absorption wavelengths of the elements<sup>40</sup> were transferred into a table for computer handling (The data was converted to MS Office Excel), and arranged so that the following procedure could be applied. Afterward we followed the steps shown below:

a) In order to make sure that the data could be handled as a set of integers and to be able to find a **G r e a t e s t Common Divisor (GCD)** different from unity ( $\text{GCD} \neq 1$ ) for that set, the column corresponding to the wavelengths (in units of  $\text{A}^\circ$ ) was multiplied by  $10^{24}$ . Different factors were tested until we found the one suiting the requirement of  $\text{GCD} \neq 1$ .

b) The GCD for the list obtained above in a) was calculated.

c) Then the list was divided in sub-lists increasing the number of lines in the sub-lists in one hundred lines, and the GCD for each sub-list was calculated. We obtained seventeen GCD values, many of which were equal. Thus we obtained a list of four different GCD values (The values found are:  $2.748779 \times 10^{11}$ ,  $4.294967 \times 10^9$ ,  $1.677722 \times 10^7$ , and  $4.194304 \times 10^6$ ). The reason to do that was that having a list of GCD, all with values way below the lowest value in the list of values of wavelengths, if we could find a GCD for this list of GCD's of the original lists, this has the highest chance of been the lowest possible common factor of all the wavelengths experimentally found ( $\delta'$ ).

d) The GCD of the list of GCD's corresponding to the sub-lists was calculated. The value obtained was 4.0; since this is the value obtained from the list after multiplying it by  $10^{24}$ , the final value of  $\delta'$  is  $\delta' = 4.0 \times 10^{-24} \text{ A}^\circ$ .

e) Following the EMG Theory of the Photon<sup>39</sup> of the photon mentioned above, we proceeded as follows:

The distance jumped by the photon is  $\Delta \mathbf{X} = \lambda/8 \rightarrow \lambda = 8 \Delta \mathbf{X}$

Expressed in fundamental units of distance  $\Delta \mathbf{X} = \mathbf{m}\delta$

Then  $\lambda = 8 \mathbf{m}\delta$ . Therefore  $\lambda_i = 8 \mathbf{m}_i\delta$ .

This implies that all wavelength values are divisible by  $8\delta$ . Therefore  $8\delta$  is the lowest possible value of a photonic wavelength (when  $\mathbf{m} = 1$ ). This value must be equal to the lowest GCD found in d), we will call this value  $\delta'$ . Once we know  $\delta'$ , when we divide it by 8 we obtain the value of  $\delta$ , the fundamental unit of distance; that is  $\delta = \delta'/8$ .

$$\delta' = 4.0 \times 10^{-24} \text{ A}^\circ = 4.0 \times 10^{-34} \text{ m}$$

$$\delta = \delta'/8 = 5.0 \times 10^{-35} \text{ m} = 5.0 \times 10^{-33} \text{ cm}$$

$$\delta = 5.0 \times 10^{-35} \text{ m} = 5.0 \times 10^{-33} \text{ cm}$$

We then can calculate the fundamental time ( $\tau$ ) from here, knowing that the speed of light in vacuum is an universal constant;

$$\mathbf{C} = \delta/\tau \rightarrow \tau = \delta/\mathbf{C}$$

Then;

$$\tau = 1.6667 \times 10^{-43} \text{ sec.}$$

The fundamental quantum of area would then be,

$$\alpha = (3/8)3^{1/2}\delta^2 \rightarrow \alpha = 1.6238 \times 10^{-65} \text{ cm}^2$$

While the fundamental volume would be,

$$\varphi = (1/8)3^{1/2}\delta^3 \rightarrow \varphi = 2.7063 \times 10^{-98} \text{ cm}^3$$

It is surprising how close the value of  $\delta$  came to the one calculated with quantum and relativist considerations, but assuming space to be a continuum.

## Conclusions

It is simple to realize that if there was only one single basic stuff the relational space would be void and nothing of what we recognize (no matter whether it is real or imagined) would have any existence. So we can certainly conclude that the basis of matter is manifold and because of that space exists.

We conclude that not all geometrical shapes would suit the necessary requirements to be the unit cell of a quantized physical space topology. This has been thoroughly demonstrated in this paper. In this theory, the Euclidean geometrical shapes of the unit cells do not represent unit volumes of space that could be filled with something, but the required topology of the unit cells for avoiding the types of contradictions already discussed in the paper. So this concept of unit cell does not bear any relation with the cubic unit space of the Heisenberg lattice theory.

If the topology of the physical space is that of a Tetrahedral Lattice, in virtue of the theory of space here proposed, we may conclude that 1) there are four fundamental interactions (the vertices of the tetrahedron), and 2) that at the fundamental level, interactions occur in groups of three (the faces connecting the tetrahedrons in the lattice).

We may define "dimension of a space" as "the number of degrees of freedom of a free object in space that cannot be expressed as a linear combination of a lesser number of degrees of freedom in the same space". In other terms, the dimensionality of a space is the base of that space. It is clearly seen that Euclidian space has three dimensions; while the Tetrahedral Lattice Space (TLS) has four dimensions, because each direction at every position is independent of the other and cannot be expressed as a linear combination of any set composed of the others. Two very important aspects of this concept of dimensionality of space are, 1) that it is not equivalent to the number of independent variables necessary to describe an object or phenomenon in that space; and 2) there is an impossibility of motion in a straight line in TLS space because the possible directions of motion between two adjacent positions in this space are the faces of the two adjacent tetrahedrons, which diverge in different angles.

A tetrahedral lattice at the macroscopic level will look three dimensional and continuous (and it will work perfectly for macroscopic phenomena), but when we want to study fundamental phenomena and entities, TLS topology would be the correct tool.

The interpretation that the emission and absorption spectra wavelengths of the elements represent naturally occurring distances, allows us to use them to calculate the smallest distance, or quantum of distance  $\delta$ . The fact that those wavelengths have been measured by a variety of laboratories around the world over a long period of time, gives us a sense of confidence on the accuracy and honesty of the values published, and thus on the resulted calculations.

The calculated value of the quantum of distance, determined empirically, using concepts of discrete space developed by the author, and presented for the first time in this paper, has been found to be  $\delta = 5.0 \times 10^{-35}$  m ( $5.0 \times 10^{-33}$  cm). This value

is so closed to the so called Planck Length ( $1.616252 \times 10^{-33}$  cm), which has been determined as an approximation (assuming space as continuous, but under quantum mechanical and relativistic energy constrains), that the strength of arguments behind these facts forces us to conclude that  $\delta$  is the fundamental distance, while the Planck Length is an approximation. But also these facts point to the conclusion that the theory presented in this paper is a strong one, because it is the only discrete space theory published so far that produces an empirically measurable parameter, whose measured value is so close to the approximated value advanced by the most fundamental physical theories of our time.

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