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Forecast Population ageing. A Comparative Study between France and Japan

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Abstract

France and Japan are countries that have had a high economic and health development over time on worldwide. This high development is the result of success public policies that these countries have implemented through time. The effect of their development has improved the health of their inhabitants particularly among the ageing persons who each time live more and more years. However, it arise the following questions due to the humans cannot live forever: what is the maximum number of years that can live the ageing persons? The maximum is the same for Japan and France? How many years France and Japan will reach that maximum? In order to answer these questions, we use the Stable Bounded Theory and the Human Mortality Database of the Berkeley University. The major results show that nowadays Japan and France are the countries that have the highest life expectancy at aging with 21.91 and 21.23 years respectively. Also is indicated the maximum for Japan that is 22.94 years and for France is 27.46 years. After that, we deduce a mathematical function that allowed us forecast the life expectancy at ageing until year 2050. The mathematical function that we found was the logistic function, such that, the upper bound of this function was the estimated maximum value. According to the results of the projecting we estimated for Japan that in 2025 the life expectancy at ageing will be 22.31 years, in 2035 will be 24.28 and in 2050 will arrive to 25.55 years. In France, the life expectancy at ageing in 2025 will be 23.13 years, in 2035 will be 24.28, and in 2050 will be 25.55. That is, Japan will reach practically the maximum in 2035, whereas France still in 2050 will be very far of the maximum value.

Keywords: Aging, economic development, social development, forecast population, stable bounded theory

Introduction

In 2010 was estimated world population of 524 million of persons. By 2050, this number is expected to be almost 1.5 billion, this is, nearly triple. During this period, the number of older people in less developed countries is projected to increase more than 250 percent, compared with just a 71 percent increase in developed countries, this means, populations will be more aging each time (World Health Organization, without/year). This world aging will intensify the financial and care burden on the working-age population in all the world´s countries (Reynolds, 2017).

This world aging will occur because the persons will live each time more and more years, implying since point of view demographic also an increase in the life expectancy in all ages. However due to humans cannot live forever, the increase in life expectancy must stop, it cannot continue growing and growing. Since mathematic point of view this means life expectancy has a maximum with respect on time, but in a colloquial way means the number of years that persons will live has a ceiling, such that, nobody can live beyond of what indicate this top limit. According to Koettl (2016) Nature has given every species an intrinsic maximum or ceiling. If every member of a species is each time healthier and healthier, life expectancy of that species will constantly increase, but eventually it will be bound by that maximum. Every species has a different ceiling. For flies it is just a couple of days, for bowhead whales it is 200 years. For humans they have found that the maximum is average 89 years. This means if we continue improving and improving the health systems, the world population´s life expectancy will converge to that maximum.

But in the case of a person who has survived until 65-years old we do not know what the maximum is. The objective of this paper is to determine that number of years for Japan and France, nowadays the two countries with the expectancy at aging highest in the entire world, according to The Human Mortality Database from the Berkeley University.

Objectives/Purpose of the study

1. Estimate the maximum number of years that can live the ageing persons in France and Japan
2. Determine if the maximum number is the same for Japan and France
3. How many years France and Japan will reach that maximum?

Methodology

Data used

We define the life expectancy at aging as the life expectancy at age 65-years old, and we will denote it with the Greek lambda. The data we used is the life tables both sexes of decadal periods of the Human Mortality Database from Berkeley University. In table 1, we can see the 10 countries that have the greatest life expectancy at aging in wide world[1]. As we can see, Japan and France have the highest life expectancy at aging in worldwide with values 21.91 and 21.23 years, respectively. In table 1 is also the life expectancy at birth. We can observe that Japan have also the biggest life expectancy at birth from all countries and France is among the eight biggest.

[1]Although the Human Mortality Database is not a world base, since, just include 39 countries or areas, due to the life expectancy is strong correlated with economic development of the countries we consider that these countries or areas are representatives of all the world, since are included in the base, the countries with the greatest economic development in worldwide.

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Table 1. The ten countries that have the greatest life expectancy at aging in worldwide

Num.	Country	Year	λ	e_0
1	Japan	2014	21.91	83.75
2	France	2015	21.23	82.15
3	Spain	2014	21.07	82.89
4	Asutralia	2014	21.02	82.63
5	Italy	2014	20.83	82.94
6	Canada	2011	20.48	81.66
7	Israel	2014	20.48	82.29
8	New Zealand	2013	20.35	81.64
9	Sweden	2016	20.29	82.34
10	Switzerland	2014	20.29	83.09

Source: Human Mortality Database

In order to get annual series we considered the middle year of the period, if period had year middle, but when the period had no year middle we did it other way. In table 2, we can see the results. Japan starts in 1948 and ends in 2012 because first period is 1947-1949 and final period is 2010-2014. France starts in 1817 and ends in 2012, because their periods are 1816-1819 and 2010-2015.

Table 2. Life expectancy at aging in Japan 1948-2012 and France, 1817-2012

Japan		France	
Year	Life expectancy at aging	Year	Life expectancy at aging
		1817	10.96
		1825	11.13
		1835	10.69
		1845	10.91
		1855	10.66
		1865	10.83
		1875	10.77
		1885	11.27
		1895	10.84
		1905	10.95
		1915	11.16
		1925	11.67
		1935	12.1
1948	11.93	1945	12.37
1955	12.63	1955	13.8
1965	13.49	1965	14.66
1975	15.21	1975	15.55
1985	17.4	1985	16.93
1995	19.87	1995	18.53
2005	20.89	2005	19.95
2012	21.51	2012	21.14

Source: Human Mortality Database

The economic and health development in France and Japan

France

After World War II, France implemented an economic policy characterized by control of certain key industries, as transportation, energy and communications, and created a planning institution to regulate economic activity. In addition, it began the construction of a welfare state and established key institutions such as social security and works councils that still currently remain. France's economic post-war policy proved to be successful and the country entered a period of accelerated economic growth, experiencing high gains in productivity, Gross Domestic Product (GDP) and real wages (France Economy, 2017).

As for the volume of GDP, France is among the 10 most important economies in wide world, whereas according to the Human Development Index, the French people have good quality of life. By other hand, if we analyze the Corruption Perception Index we see that the population in France has a low level of perception of government corruption (France: Economy and demography, without year).

Besides, France is the first tourist destination in the wide world. In 2011, it received almost 80 million foreign tourists representing 6.4 per cent of its GDP. In 2010, more than 61 per cent of their agricultural and viticulturally products were sent to the member states of the European Union, as well as, United States and China, what locates it as the second largest exporter on this subject. Likewise, the country is fifth importer in the world, having as commercial partners Germany, Belgium, China, Italy, Spain and the United States (Kolodziejski, 2012).

In manufacturing, France is one of the country's leaders in the automotive, aerospace and railway sectors, cosmetics and luxury goods. Furthermore, it has a highly educated labor force and the highest number of science graduates per thousand workers in Europe. Primary exports are machinery and transportation equipment, aerospace equipment and plastics, while primary imports include machinery, automobiles and crude oil (France Economy, 2017).

Since the 1980s, the economic policy of France has favored capitalism and market-orientated policies. The government privatized partially or fully many national industries, as Air France, France Telecom and Renault, and today France's leaders remain committed to capitalism. However, the French government still plays a role in certain key national sectors, such as agriculture, and it will intervene in the market to moderate certain social economic inequalities (France Economy, 2017).

The French Health System was ranked No. 1 according to World Health Organization in 2000. It has 2–5 indicators of overall satisfaction and health status supports this ranking. The French health system can be considered between the Britain Health System where there is too much rationing and the United States Health System where too many people have no health insurance. In last 3 decades, French successive governments have been worried with health care reform. Since 1996, whether governments of left or right have pursued cost control policies without reforming the overall management and organization of the health system (Rodwin, 2003).

After more than a half century of struggle, in January 2000 France insured remaining 1 per cent of its population that was no insured and offered supplementary coverage to 8 per cent of its population with a below an income ceiling low. This extension of health insurance do that France is an interesting case of how to ensure universal coverage with a incremental reform while maintaining a sustainable system that limits perceptions of health care and restrictions on patient choice (Rodwin, 2003).

The French Health System combines universal coverage with a public-private mix of hospital and ambulatory care, higher levels of resources and a higher volume of service provision than in the United States. It has a wide access to comprehensive health services for a population that is on average older than that of the United States having expenditures were equal to 9.5% of its GDP compared with 13.0% of GDP in the United States (Rodwin, 2003).

Japan

After the World War II, Japan´s economy was developed based on an industrial infrastructure very destroyed during the war. In 1952, at end of the allied occupation, Japan was considered a less-developed country, with a per capita consumption almost one fifth that of the United States. However, during the following two decades Japan grew to 8 percent rate average annual, enabling become in the first country that was moved to developed status. In the 1950-1970 period the percentage of Japanese that lived in cities grew of 38 per cent to 72 percent, increasing the industrial work force. After that, during the decade of 1960s the exports grew on average 18.4 percent annual. After 1965 Japan achieved a surplus in the current account balance every year. The economic growth of this epoch was accompanied by important changes in Japan´s industrial structure. The iron and steel, shipbuilding, machine tools, motor vehicles and the electronic devices came to dominate the industrial sector in wide world (Facts and Details, without year).

The competitive strength of Japanese industry increased steadily, with exports growing on average 18.4 percent per year during the 1960s. After the mid-1960s the economic growth, supported by strong private-sector facilities investment based on a high personal savings ratio, was accompanied by significant changes in Japan's industrial structure. Whereas formerly the mainstays of the economy were agriculture and light manufacturing, the focus shifted to heavy industry. Iron and steel, shipbuilding, machine tools, motor vehicles, and electronic devices came to dominate the industrial sector. Government economic planning aimed at expansion of the industrial base proved exceedingly successful, and by 1968 national income had doubled, achieving an average annual growth rate of 10 percent (Japan Fact Sheet, without year).

Throughout the postwar period, Japan's economy continued to boom, with results far outstripping expectations. Japan rapidly caught up with the West in foreign trade, GDP and general quality of life. These achievements were underscored by the 1964 Tokyo Olympic Games and the Osaka International Exposition world's fair in 1970.

Nowadays, Japan is the second economy in wide world according the volume of its GDP and is the country with the lowest unemployment rate with only 2.7 percent of its population and in terms of the Human Development Index the data indicate the Japanese have a good quality of life (Japón: Economia y Demografía, without year).

Currently, the health system in Japan has one of the highest levels in the world in many facets. The amendment to the National Sickness Insurance Act of 1961 granted the right to all Japanese citizens and foreign residents to receive benefits from one of the six existing health insurance programs in Japan. The Japanese health insurance system establishes that the 20 percent of medical expenses are responsibility of the beneficiaries if they are newborns to preschool children; 30 percent for primary school children up to 69 year olds; and 20 percent for people over 70 years of age. To the older than 75 years of age, the Longevity Medical Care System is applied, which is different from the general system (Japan Fact Sheet, without year).

This system of universal medical assistance provides all citizens with adequate health care, and in this way contributes significantly to the level of health throughout Japanese society. In 2005, an amendment to the Long Term Care Insurance Act placed emphasis on prevention, with the aim of help people with relatively mild problems to maintain and improve their health, and thus prevent them from deteriorating to the point where it becomes necessary to provide more comprehensive care (Japan Fact Sheet, without year).

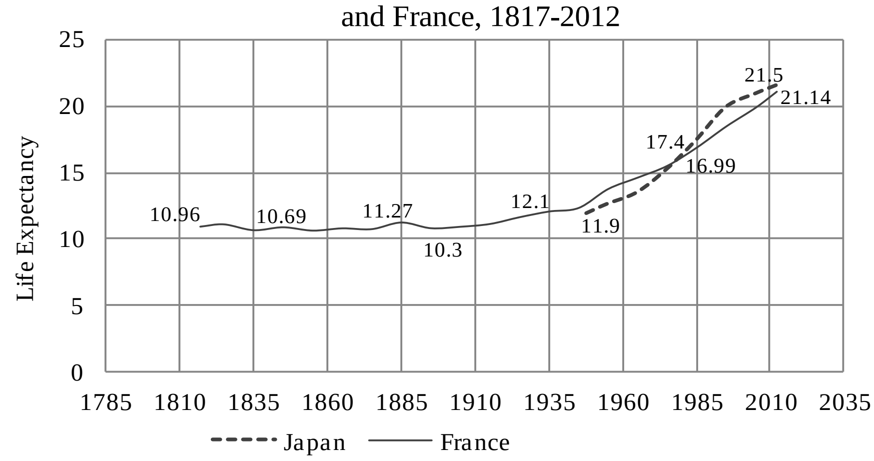
The Government created a long-term care insurance in 2000, which collects compulsory contributions from a broad sector of the population to provide services such as home visits of domestic helpers, the possibility of going to assistance centers or extended stays in residences, of elderly people for people suffering from senile dementia or who cannot get out of bed because of health problems (Japan Fact Sheet; without year).

As we can realize France and Japan have high levels of economic and health development. Their indicators in this mater are the highest in the world. This development has contributed to improve the global health of their inhabitants, decreasing nowadays the child mortality and increasing both the life expectancy at birth and the life expectancy at aging.

The evolution of the life expectancy at aging in Japan and France at last years

In figure 1, we can see the life expectancy at aging of Japan at last 64 years and in France at last 195 years. During this time, Japan increased 9.6 years its life expectancy at aging, passing of 11.9 years in 1948 to 21.51 in 2012. In France, we can identify two phases which have almost 100 years each one. The first phase was between 1817 and 1905, and the second between 1905 and 2012. As we can observe the first was of stability, whereas the second was of growing. In the phase of stability there was not increase, life expectancy at aging was keep around the 10.96 years, but in the growing phase it arrived until 21.14 in 2012, representing a growing of 10.2 years with respect to the stability phase.

Figure 1. Life expectancy at aging in Japan 1948-2012 and France, 1817-2012



Source: Human Mortality Database

In figure 1, we can also compare the evolution of life expectancy at aging in the two countries starting of 1948. As it can see, life expectancy at aging in Japan was below of France during 1948-1975 period, this is, during 27 years. But after 1975 until the present the life expectancy in Japan has been greater than in France. In 1975 in Japan was 17.4 years and in France of 16.99, and for 2012 in Japan was 21.5 and in France of 21.14. However, we can also observe that in last seven years the growing speed of the life expectancy at aging in Japan has gone down. According the data, in last seven years the growing speed in Japan was 0.09 years of live per year, whereas in France was 0.17 respectively, what means in a few years the life expectancy at aging in France will be greater than in Japan.

As we can see, according to the data trends the life expectancy at aging at both countries still will continue growing, although in France with a greater fast than in Japan, so that, now the questions are, 1) what is the maximum value of the life expectancy at aging for Japan and France?, and 2) in what year they will be reached. The answers regarding its existence and about calculation of these constants seem to be in the Stable Bounded Theory (Gonzalez-Rosas, 2012).

The maximum of the life expectancy at aging in Japan and France

The Stable Bounded Theory rest in two fundamental postulates, first, in each year the life expectancy at aging is a random phenomenon, so, according to the probability theory, in each year must have a mean and a variance. Second, the life expectancy at aging is equal to a mathematical function which depends on time. These two postulates imply that in each year the observations of life expectancy at aging will be equal a quantity determined by the mathematical function plus a certain quantity that cannot be determined because is random. The mathematical function is called the deterministic component and the quantity that cannot be determined the stochastic component (Medhi, 1981). So, under these postulates the behavior equations of the observations and the mean of life expectancy at aging in each time would be:

$$\lambda_t = f(t) + \varepsilon_t \tag{1}$$

$$\mu_t^{\lambda} = f(t) + \varepsilon_t \tag{2}$$

Where:

λ_t denotes the observation of life expectancy at aging on time t,

$f(t)$ is a mathematical function unknown,

ε_t are random variables on time t that are supposed independent, with Normal probabilistic distribution, mean equal to zero and constant variance, and

μ_t^{λ} denotes the mean of the life expectancy at aging on time t.

In order to prove that maximum exists, the Stable Bounded Theory uses the change amount of life expectancy at aging with respect of time. Due to, the change amount between a time and other is measured with the slope of the straight line that joins two points of the bi-dimensional space defined by the figure 1, we calculated the slopes and the middle values[2] between two consecutive values of the life expectancy at aging of following way:

$$\nabla_i^{\lambda} = \frac{\lambda_{t+1} - \lambda_t}{t+1-t} \tag{3}$$

$$MV_i = \lambda_t + \frac{\lambda_{t+1} - \lambda_t}{2} \tag{4}$$

Where

∇_i^{λ} Denotes the slope of the straight line between the points (t, λ_t) and $(t+1, \lambda_{t+1})$ of the bi-dimensional space defined by figure 1 (Leithold, 1973, p.137), and

MV_i Represents the middle value between the values of the life expectancy at aging denoted like λ_t and λ_{t+1}.

For Japan, table 3 presents the results and in figure 2, on X-axis are the values of the life expectancy at aging and on the Y-axis the slopes values. As we can see, the points are not exactly on the dotted line, it means the behavior of the slopes is also random, such that, according to the probability theory, it will also have a mean and a variance. In figure 2, the mean of the slopes is represented by the dotted line, which it seems to be a parabola and we will denote with the letter *g*. It is important also point out that *g* depends of the life expectancy at aging, so, we will denote this fact as $g(\lambda)$.

[2] The *Stable Bounded Theory* proves that exist three estimators of the maximum value. One of them is associated with λ_t, other with λ_{t+1}, and one more with the middle value between the two. The Theory also proves that the best of the three is middle value.

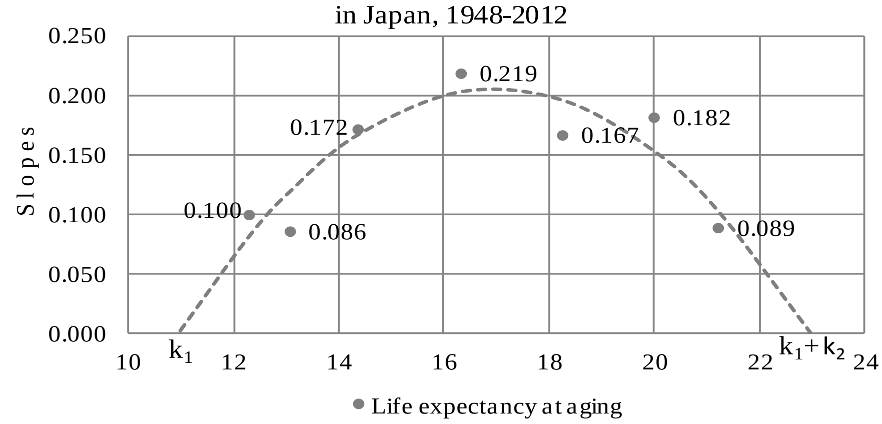
Table 3. Life expectancy at aging, middle values and slopes in Japan, 1948-2012

Year	λ	Middle values	Slopes
1948	11.93	12.28	0.100
1955	12.63	13.06	0.086
1965	13.49	14.35	0.172
1975	15.21	16.31	0.219
1985	17.4	18.24	0.167
1995	19.07	19.98	0.182
2005	20.89	21.2	0.089
2012	21.51		

Source: Calculations based on equations 3 and 4.

If the assumption of the parabola is true then must exist two values of the life expectancy at aging where the curve of function g intersect the X-axis. In those values, denoted in figure 2 as k_1 and k_1+k_2 the slope is zero, what implies that the mean of life expectancy at aging will have in future a maximum, but also imply that had a minimum in the remote past.

Figure 2. Slopes and middle values of life expectancy at aging in Japan, 1948-2012



Source: Table 3

In order to prove mathematically the hypothesis about the parabola and hence existence of the maximum and minimum we adjusted a regression model (Johnston, 1972) to the data of figure 2, with the slope as the variable dependent and life expectancy at aging like independent variable, that is,

$$\nabla_J = A\lambda_J^2 + B\lambda_J + C + \omega \quad (5)$$

$$\mu_J^{\nabla} = A\lambda_J^2 + B\lambda_J + C \quad (6)$$

Where

∇_J, denotes the random variable of the slope in Japan,

λ_J, denotes the life expectancy at aging in Japan,

A, B and C, are unknown constants,

ω, are random variables that are supposed independent, with Normal probabilistic distribution, mean equal to zero and constant variance, and

μ_J^{∇}, denotes the mean of the slope of the life expectancy at aging on time t in Japan. According to Medhi (1981) in equation 5 the deterministic component is $\mu_J^{\nabla} = A\lambda_J^2 + B\lambda_J + C$ and the stochastic component is ω. From the mathematical point of view the maximum and minimum values are those values of the life expectancy at aging where the deterministic component is zero, namely,

$$A\lambda_J^2 + B\lambda_J + C = 0$$

Thus using the formulas to calculate the roots of a parabola, we have:

$$k_1 = \frac{-B}{2A} + \frac{\sqrt{B^2 - 4AC}}{2A} \quad (7)$$

$$k_1 + k_2 = \frac{-B}{2A} - \frac{\sqrt{B^2 - 4AC}}{2A} \quad (8)$$

These results indicate that formulas 7 and 8 are estimators of the minimum and maximum of the life expectancy at aging respectively.

In order to get the best estimators of the constants A, B and C and hence the best estimators of the maximum and minimum we applied the ordinary least squares method to the data of table 3. Table 4 presents the estimations and the p-values to determine their statistical significance.

Table 4. Estimated constants of the equation 6 and p-values

Constant	Estimate	Standard error	t-value	p-value
A	-0.0057	0.00153	-3.72	0.02
B	0.1933	0.05126	3.77	0.02
C	-1.4348	0.41517	-3.46	0.026

As we can see, the p-values of the three constants are less than 0.05, therefore, they are significantly different of zero (Johnston, 1972), and so, to estimate the maximum and minimum values we substitute the estimations of the coefficients A, B and C, in 7 and 8 equations,

$$k_1 = \frac{-0.1933}{2(-0.0057)} + \frac{\sqrt{0.1933^2 - 4(-0.0057)(-1.4348)}}{2(-0.0057)}$$

$$k_1 = 10.97$$

$$k_1 + k_2 = \frac{-0.1933}{2(-0.0057)} - \frac{\sqrt{0.1933^2 - 4(-0.0057)(-1.4348)}}{2(-0.0057)}$$

$$k_1 + k_2 = 22.94$$

And k_2 is estimated as

$$k_2 = 22.94 - 10.97$$

$$k_2 = 11.97$$

In addition to the statistical significance of the constants we find the F Statistic value was 7.36 with a p-value of 0.0462 which is less than 0.05, what proves the parabola assumption in 5 and 6 is true. By other hand we find the determination coefficient was 78.5%, and the assumptions about the residuals of 5 are also true (Montgomery and Peck, 1982). These results mathematically prove that the life expectancy at aging in Japan has a maximum and a minimum.

For the case of France, in table 5 we can see the calculations of the middle values and the slopes. In figure 3 we can verify it is also a random phenomenon whose mean is a parabola function, this is,

$$\nabla_F = a\lambda_F^2 + b\lambda_F + c + \delta \tag{9}$$

$$\mu_F^{\nabla} = a\lambda_F^2 + b\lambda_F + c \tag{10}$$

Where

∇_F, denotes the random variable of the slope in France,

λ_F, denotes the life expectancy at aging in France,

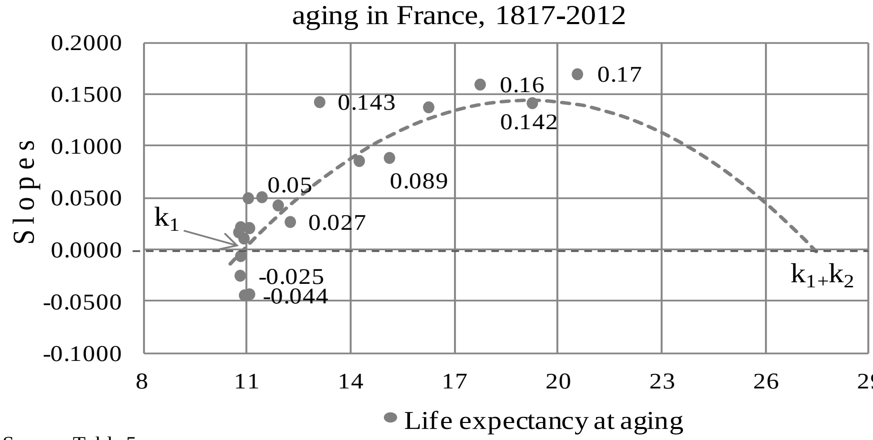
a, b and c, are unknown constants,

δ, are random variables that are supposed independent, with Normal probabilistic distribution, mean equal to zero and constant variance, and

μ_F^{∇}, denotes the mean of the slope of the life expectancy at aging on time *t* in France.

In order to estimate the constants *a*, *b* and c was adjusted a regression model. The table 6 presents the estimations of ordinary least squares method and the p-values to determine their statistical significance.

Figure 3. Slopes and middle values of life expectancy at aging in France, 1817-2012



Source: Table 5

Table 5. Life expectancy at aging, middle points and slopes in France, 1817-2012

Year	Life expectancy at aging	Middle values	Slopes
1817	10.96	11.045	0.0213
1825	11.13	10.91	-0.044
1835	10.69	10.8	0.022
1845	10.91	10.785	-0.025
1855	10.66	10.745	0.017
1865	10.83	10.8	-0.006
1875	10.77	11.02	0.05
1885	11.27	11.055	-0.043
1895	10.84	10.895	0.011
1905	10.95	11.055	0.021
1915	11.16	11.415	0.051
1925	11.67	11.885	0.043
1935	12.1	12.235	0.027
1945	12.37	13.085	0.143
1955	13.8	14.23	0.086
1965	14.66	15.105	0.089
1975	15.55	16.24	0.138

1985	16.93	17.73	0.16
1995	18.53	19.24	0.142
2005	19.95	20.545	0.17
2012	21.14		

Source: Calculations based on equations 3 and 4

Table 6. Estimated constants of the equation 8 and p-values to prove their statistical significance

Constant	Estimate	Standard error	t-value	p-value
a	-0.0021	0.00094	-2.2	0.042
b	0.0805	0.0281	2.86	0.011
c	-0.6269	0.20059	-3.13	0.006

As we can see, the p-values are less than 0.05, so the three constants are significantly different of zero. In this way, to estimate the maximum and minimum values for France we substitute the estimations of the coefficients a, b and c, in 7 and 8 equations,

$$k_1 = \frac{-0.0805}{2(-0.0021)} + \frac{\sqrt{0.0805^2 - 4(-0.0021)(-0.6269)}}{2(-0.0021)}$$

$$k_1 = 10.87$$

$$k_1 + k_2 = \frac{-0.0805}{2(-0.0021)} - \frac{\sqrt{0.0805^2 - 4(-0.0021)(-0.6269)}}{2(-0.0021)}$$

$$k_1 + k_2 = 27.46$$

And k_2 is estimated as

$$k_2 = 27.46 - 10.87$$

$$k_2 = 16.59$$

Besides, the F Statistic value was 35.77 with a p-value of 0.0000 which prove the parabola assumption in 5 and 6 is true. The determination coefficient was 78.54 percent, and the assumptions about the residuals of 5 are also true[3]. These results verify mathematically that also in France the life expectancy at aging has a maximum and a minimum.

Besides, the F Statistic value was 35.77 with a p-value of 0.0000 which prove the parabola assumption in 5 and 6 is true. The determination coefficient was 78.54 percent, and the assumptions about the residuals of 5 are also true. These results verify mathematically that also in France the life expectancy at aging has a maximum and a minimum.

As we can realize, the minimum values for the life expectancy at aging in France and Japan are almost equal 10.98 and 10.87 respectively. However, the maximum value for France is a lot big than in Japan, 27.46 and 22.94 respectively. The problem now is to know when those values are going to be reached them. To solving this problem, first, we find the equation of life expectancy at aging and time, and secondly, we forecast it using that equation.

The equation of life expectancy at aging and time in Japan and France

According to the postulates of the Stable Bounded Theory, the behavior equations of the observations and mean of life expectancy at aging for both countries at each time are,

$$\lambda_t = f(t) + \varepsilon_t$$

$$\mu_t^{\lambda} = f(t)$$

However, in practice the function $f(t)$ is unknown, but if we based on the data´s trend and the fact the maximum and minimum values exist, we can have an idea of how is the derivative of $f(t)$. Initially, both in Japan and in France, according to the data´s trend, $f(t)$ must be an increasing function and therefore its derivative must be positive. Furthermore, due to existence of the maximum and minimum values the derivative must be zero at those values. These properties are called differential properties. Using these properties the Stable Bounded Theory determines a differential equation.

The Stable Bounded Theory supposed that the derivative of $f(t)$ is given by the product of two functions $h_1(\lambda)$ and $h_2(t)$, this is, one depends of the life expectancy at aging and the other depends on the time, forming a differential equation of separable variables (Wilye, 1979), such that when we solve it, we have an equation of the function $f(t)$, namely,

$$\frac{df}{dt} = h_1(\lambda) h_2(t)$$

But as the derivative must be positive and zero in the maximum and minimum values, so, the functions h_1 y h_2 can be as follow

$$h_1(\lambda) = (\lambda - k_1)(\lambda - (k_1 + k_2)) \text{ and}$$
$$h_2(t) = m \quad ; \quad m < 0$$

And hence

$$\frac{df}{dt} = (\lambda - k_1)(\lambda - k_1 - k_2)\, m \tag{11}$$

Where

λ, denotes the life expectancy at aging, and

k_1 And $k_1 + k_2$ are the minimum and maximum values respectively.

From equation 11, since k_1 is the minimum of life expectancy at aging, the quantity $(\lambda - k_1)$ is always positive, and due to $k_1 + k_2$ is the maximum, the quantity $(\lambda - (k_1 + k_2))$ is always negative, therefore $(\lambda - k_1)(\lambda - (k_1 + k_2))$ is negative, but when we multiply by m which is negative, finally the derivative is positive, satisfying the first differential property. In the other hand, we can also see that if λ is equal to k_1 and $k_1 + k_2$, so, the derivative is zero, fulfilling the second differential property.

After that we solved the equation 9. First we separate variables

$$\frac{1}{(\lambda-k_1)(\lambda-k_1-k_2)}\,df = m\,dt$$

And after, we integrate

$$\int \frac{1}{(\lambda-k_1)(\lambda-k_1-k_2)}\,df = \int m\,dt$$

But, according to Gonzalez-Rosas (1988), who proved that solving by partial fractions the integral of the left side and clearing the λ variable, finally we have that,

$$\lambda = k_1 + \frac{k_2}{1+e^{\alpha+\beta t}} \quad ; \quad \beta < 0 \qquad (12)$$

Where λ denotes the life expectancy at aging, k_1 and $k_1 + k_2$ denote the minimum value and the maximum value respectively, and α and β are constants that determine how quickly the life expectancy at aging approaches to the maximum value, Gonzalez-Rosas (2017) called these constants the parameters of the fastness. It is very important to make clear that deduction of equation 10 is the same, both France and Japan, only the constants are going to change. The equation 10 is known like the logistic function and is used commonly in the demography to projecting the demographic phenomena in this case we used it, to projecting the life expectancy at aging in Japan and France. To carry out the projecting we had to estimate the parameters of the fastness for both countries.

Estimation of the parameters of the fastness

According to Draper and Smith (1966) equation 10 is no linear at the constants α and β, so, they cannot be estimated by the method of least squares. However, applying algebra we find that,

$$\ln\left(\frac{k_2}{\lambda-k_1}-1\right) = \alpha + \beta t \qquad (13)$$

The result is an equation linear at the constants α and β, what imply that now they can be estimated by ordinary least squares method or generalized least squares. This suggests that estimation of the constants of the equation 10 can be done in two stages. First, are estimated k_1 and k_2, and secondly, α and β. Gonzalez-Rosas (2017) has called to the variable $\ln\left(\frac{k_2}{\lambda-k_1}-1\right)$ the transformed of the life expectancy at aging.

In case of Japan we substituted $k_1 = 10.97$ and $k_2 = 11.97$ in equation 11 and we calculated the transformed of the life expectancy at aging (see table 7), and in figure 4, we can check that, the relation between the transformed of the life expectancy at aging and time is given effectively by a straight line, as is predicted by the equation 11, so that, in order to estimate the constants α and β we adjusted a simple regression model to data of figure 4.

Table 7. Time, life expectancy at aging and the transformed in Japan, 1948-2012

Year	Time	Life expectancy at aging	Transformed
1948	0	11.93	2.44
1955	7	12.63	1.83
1965	17	13.49	1.32
1975	27	15.21	0.6
1985	37	17.4	-0.15
1995	47	19.07	-0.74
2005	57	20.89	-1.58
2012	64	21.51	-2

Source: Time was calculated as year-1948 and the transformed based on equation 11

Figure 4. Transformed of the life expectancy at aging and time in Japan, 1948-2012

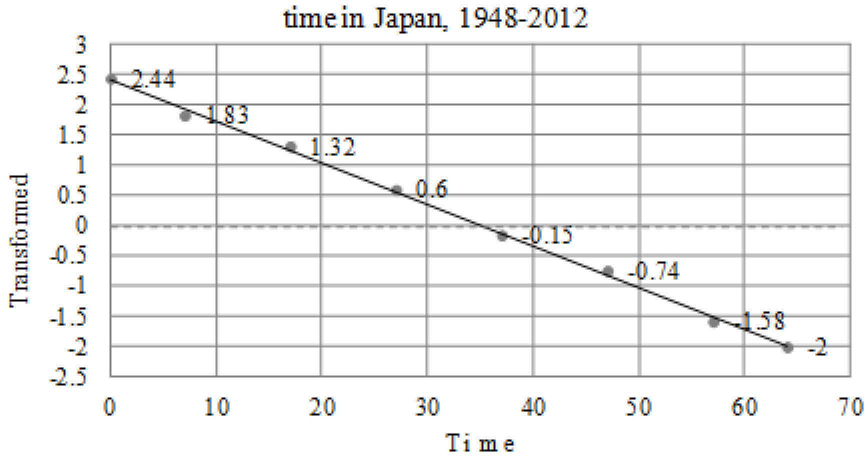


Table 8, presents the estimates of ordinary least squares of the constants and the p-values to prove their statistical significance. Note that both constants are statistically significant with values $\alpha = 2.4223$ and $\beta = -0.0689$.

For France, we substitute $k_1 = 10.87$ and $k_2 = 16.59$ in 11 and the results are presented in table 9. In figure 5, we can check again that, the relation between the life expectancy at aging transformed and time is given effectively by a straight line, so that, in order to estimate the constants α and β we adjusted a simple regression model to the data of figure 5. In table 9, we can observe that time begin in 1925 not in 1817, because the equation 11 is not true in the stable state, that is, if time begin in other year more back of 1925 the relation between the transformed of the life expectancy and time, is not a straight line.

In figure 1, we can see that 1925, the life expectancy at aging begins to increase, that is, in that year, it begun to be broken the stability.

Table 10, presents the estimates of ordinary least squares of the constants and the p-values to prove their statistically significant. According to the results, both constants are significant with $\alpha = 2.9514$ and $\beta = -0.0399$.

Table 8. Estimated constants of the equation 11 and p-values to proving their statistical significance in Japan

Constant	Estimate	Standard error	t-value	p-value
α	2.4223	0.04489	53.96	0.000
β	-0.0689	0.00116	-59.49	0.000

Table 9. Time, life expectancy at aging and the transformed in France, 1925-2012

Year	Time	Life expectancy at aging	Transformed
1925	0	11.67	3.0207
1935	10	12.1	2.5494
1945	20	12.37	2.3288
1955	30	13.8	1.5498
1965	40	14.66	1.2250
1975	50	15.55	0.9405
1985	60	16.93	0.5575
1995	70	18.53	0.1573
2005	80	19.95	-0.1865
2012	87	21.14	-0.4826

Source: Time was calculated as year-1925 and the transformed based on equation 11

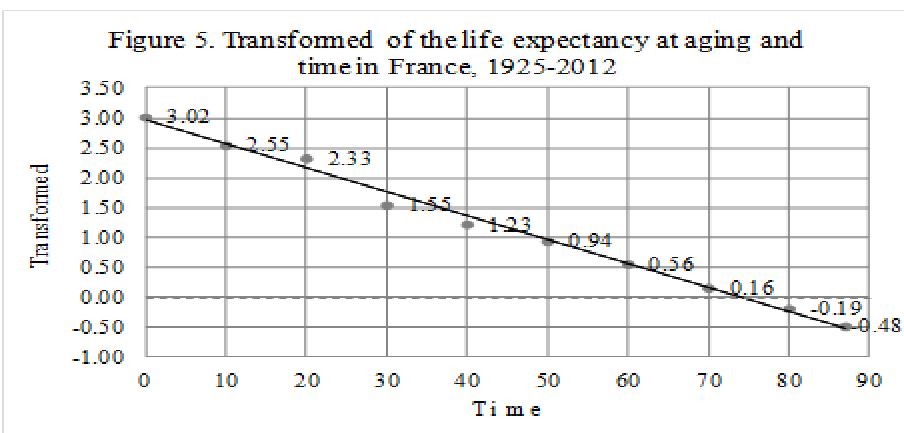


Table 10. Estimated constants of the equation 11 and p-values

Constant	Estimate	Standard error	t-value	p-value
α	2.9514	0.06581	44.84	0.000
β	-0.0399	0.00124	-32.1	0.000

With the estimate of the parameters of the fastness the equations of the behavior of the observations and of the mean of the life expectancy at aging in Japan and France were completely solved.

The equation of the observations can be used to elaborate projections by interval, whereas the equation of the mean can be used to carry out punctual forecasts of the life expectancy at aging.

Results

Forecast of the life expectancy at aging for Japan and France

The above results prove that the behavior of the mean of life expectancy at aging through time in Japan and France are governed by following mathematical equations

$$\mu_{\lambda}^{J}(t) = 10.97 + \frac{11.97}{1+e^{2.422-0.069t}} \quad (14)$$

$$\lambda_{\lambda}^{F}(t) = 10.87 + \frac{16.59}{1+e^{2.9514-0.0399t}} \quad (15)$$

Where

μ_{λ}^{J} And μ_{λ}^{F},denote the mean of the life expectancy at aging in Japan and France in time t respectively,

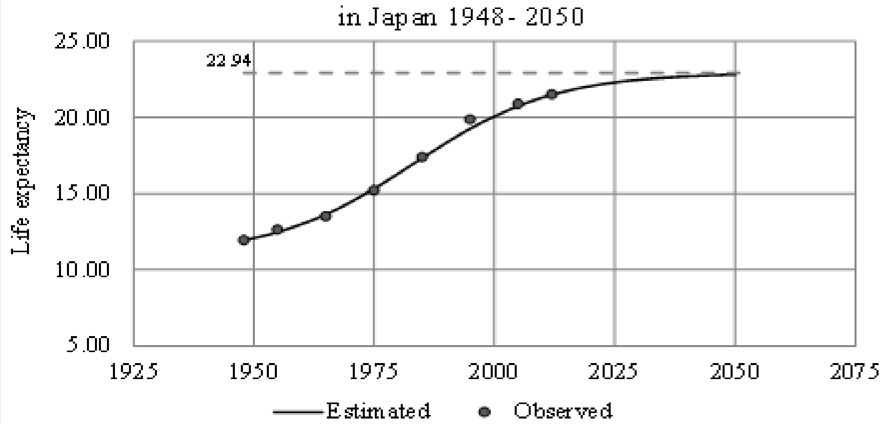
10.97 And 10.87 represent the minimum values of the life expectancy at aging in Japan and France respectively, $10.97+11.97$ And 10.87+16.59 are the maximum values of the life expectancy at aging in Japan and France respectively,

It is important to make clear what we are going to forecast is the mean and not the observations of the life expectancy at aging, because the observations cannot be predicted, since they are random variables and the random phenomenon cannot be forecast. So that, when we gave values to time variable in equation 14 we obtained forecasts of the life expectancy at aging for period 2013-2050 in Japan.

For France, we obtained forecasts using equation 15.

In Japan, we can observe by figure 6, the model is adjusted very well to the observed data. According to the results of the equation 12, we found that in next 38 years the life expectancy at aging in Japan will growing very little, only will increase 1.31 years, due to almost will remain stable since 2035. In 2025 will be 22.31 years, in 2035 increase to 22.62 and in 2050 will arrive to 22.82 years, very near of the maximum value 22.94 years.

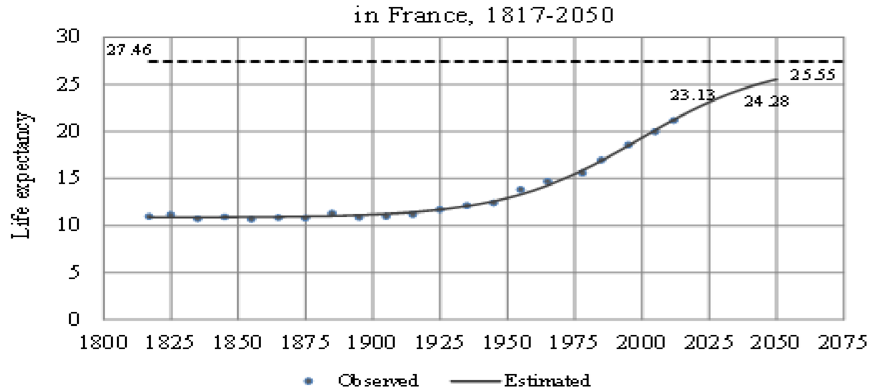
Figure 6. Life expectancy at aging observed and estimated in Japan 1948- 2050



Source: Table 3 and estimates based on 12

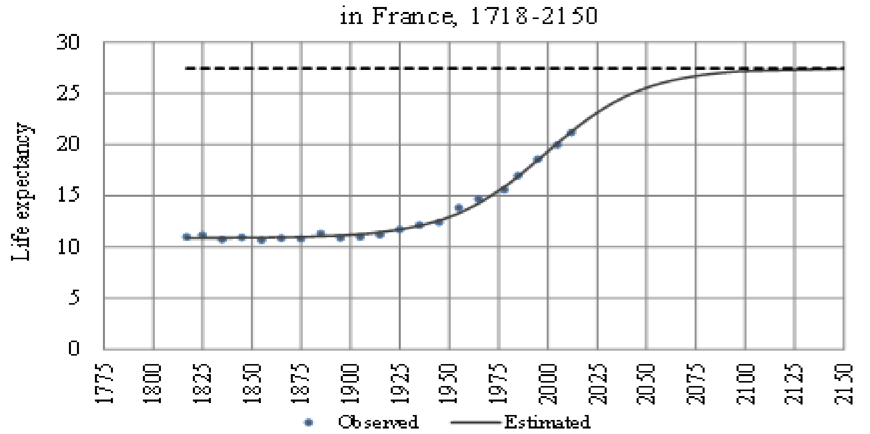
In France the model is also adjusted very well to the data observed (figure 7). But the situation will be very different respect of Japan. First, the growing will be of 4.41 years in next 38 years, and following, no yet it will not have been reached the stability. In 2025 the life expectancy at aging will be 23.13 years, in 2035 of 24.28 years, and in 2050 will reach the 25.55 years, almost 2 years by under the maximum value which is 27.46. If we extend time beyond 2050 we find that the stability in France will be reached until 2125 (see figure 8).

Figure 7. Life expectancy at aging observed and estimated in France, 1817-2050



Source: Table 1 and estimations based on equation 13

Figure 8. Life expectancy at aging observed and estimated in France, 1718-2150

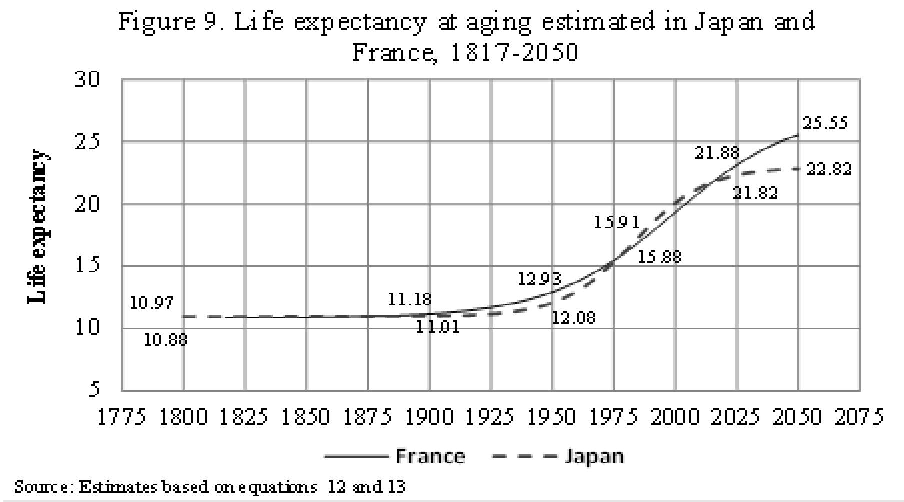


Source: Table 1 and estimates based on equation 13

In Figure 9, we can compare in both countries the evolution of the life expectancy at aging, both in past and in future. As we can observe, in 1800, that is, 217 years ago, the life expectancy at aging in Japan and in France was practically the same, with just a difference of 33 days (0.09 years) in favor of Japan.

After that, there was a stability period in both countries that remained during almost 75 years in France (around 1875) and almost 100 years in Japan (around 1900). What means that during 25 years the life expectancy at aging in France was growing whereas in Japan was stable, what had as consequence that in 1900 the life expectancy at aging in France overcome to Japan by 62 days (0.17 years). After 1900 and during almost 80 years the life expectancy at aging in France was greater than in Japan, until 1978 when Japan got over it per 11 days (0.03 years).

[3]The residuals analysis (stochastic component) indicates the random variables have a normal distribution, they are independent and they have constant variance.



The difference will continue in favor of Japan during next almost 40 years until 2016 when the life expectancy at aging in Japan will begin to stabilize again what will bring as a consequence that in 2016 once again the life expectancy at aging in France will be greater than in Japan per 22 days (.06 years). However, in 2050 whereas in Japan will be already stable, in France still will continue growing what will imply for 2050 that the life expectancy at aging in France will be greater than in Japan by 2.73 years, the biggest difference in 250 years of evolution in these two countries.

Discussion and Conclusion

When we consider the life expectancy at aging as a random phenomenon, the observations have irregular behavior which implied the hypothesis that we cannot be able to forecast the life expectancy at aging since the random phenomenon cannot be predicted. So, the question arose, how can we predict a random phenomenon?

The answer according to the probability theory is that since life expectancy at aging must have a mean and a variance, the mean have a deterministic behavior given by a mathematical function that depends on time, so, we were be able to predict at least the mean of the life expectancy at aging. However, according to data trend this function must be increasing, but life expectancy at aging cannot growing and growing, so, we supposed that it would have stabilize in a maximum value, but this situation brought us two questions, first, what is that value where the life expectancy at aging is going to stabilize in future? And secondly, what is the function that we must to use to predict it in future?

These two questions, we answered them using The Stable Bounded Theory which allowed us to prove the existence of the maximum value and besides calculate it. Also it helped us to find the function which allowed us to do the predictions of the life expectancy at aging.

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